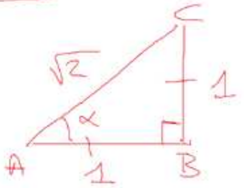
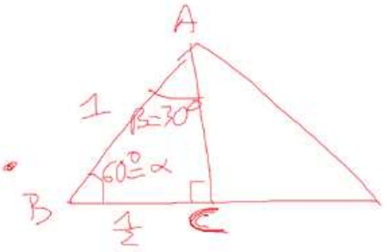
Propriété 2



$$\cos\alpha = \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\alpha = \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = 1$$



$$AC^2 = AB^2 - BC^2$$
$$= 1 - \frac{1}{4} = \frac{3}{4}.$$

donc $AC = \frac{\sqrt{3}}{2}.$

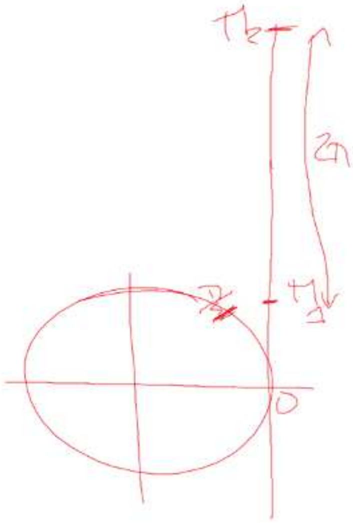
$$\cos\alpha = \cos 60^\circ = \frac{BC}{AB} = \frac{1}{2} \qquad \cos\beta = \cos 30^\circ = \frac{AC}{AB} = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \sin 60^\circ = \frac{AC}{AB} = \frac{\sqrt{3}}{2} \qquad \sin\beta = \sin 30^\circ = \frac{BC}{AB} = \frac{1}{2}$$

$$\tan\alpha = \tan 60^\circ = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \times \frac{\cancel{2}}{1} = \sqrt{3}$$

$$\tan\beta = \tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

Exemple 1



$OM_1 = \frac{\pi}{4}$

$OM_2 = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$

$OM_3 = \frac{9\pi}{4} + 2\pi = \frac{17\pi}{4}$

Donc $\frac{9\pi}{4}$, $\frac{17\pi}{4}$ et $-\frac{7\pi}{4}$ ont le même point associé sur le cercle.

3) De même $3\pi, -\pi, 1001\pi$ ont le même point associé sur le cercle.

4) Un tour correspond à $6\frac{\pi}{3}$

Or $113 = 6 \times 18 + 5$

Donc $113\frac{\pi}{6} = \cancel{6} \times 18 \times \frac{\pi}{\cancel{6}} + 5 \times \frac{\pi}{6}$

$= 18\pi + 5\frac{\pi}{6}$

Ainsi $\frac{5\pi}{6} \in [0; 2\pi]$ et $\frac{5\pi}{6}$ a le même point associé sur le cercle que $\frac{113\pi}{6}$.

6) Un tour correspond à $\frac{8\pi}{4}$.

Or $59 = 7 \times 8 + 3$

Donc $\frac{59\pi}{4} = 7 \times 8 \times \frac{\pi}{4} + \frac{3\pi}{4}$

$= 7 \times 2\pi + \frac{3\pi}{4}$.

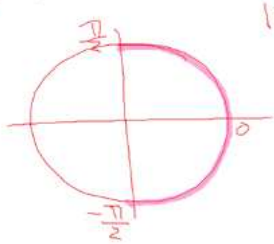
Ainsi $\frac{3\pi}{4} \in [0 ; 2\pi]$ et $\frac{3\pi}{4}$ a le même point associé sur le cercle que $\frac{59\pi}{4}$.

Exemple 2

1] On sait que : $\cos^2\alpha + \sin^2\alpha = 1$

$$\text{donc } \cos^2\alpha = 1 - \sin^2\alpha$$
$$= 1 - \left(\frac{3}{10}\right)^2$$
$$= \frac{91}{100}$$

Ainsi $\cos\alpha = \frac{\sqrt{91}}{10}$ ou $\cos\alpha = \frac{-\sqrt{91}}{10}$



Or $\alpha \in \left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]$

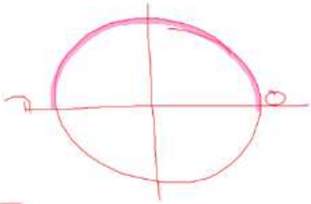
alors $\cos\alpha \geqslant 0$

Donc $\cos\alpha = \frac{\sqrt{91}}{10}$.

3] On sait que : $\cos^2\alpha + \sin^2\alpha = 1$

$$\text{Donc } \sin^2\alpha = 1 - \cos^2\alpha$$
$$= 1 - \left(\frac{3}{7}\right)^2$$
$$= \frac{40}{49}$$

Ainsi $\sin\alpha = \frac{2\sqrt{10}}{7}$ ou $\sin\alpha = \frac{-2\sqrt{10}}{7}$.

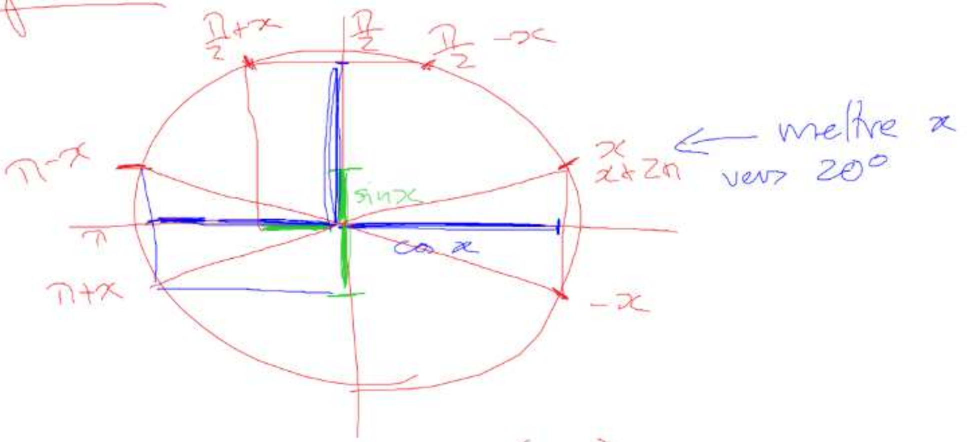


Or $\alpha \in [0 ; \pi]$

alors $\sin\alpha \geqslant 0$.

Donc $\sin\alpha = \frac{2\sqrt{10}}{7}$

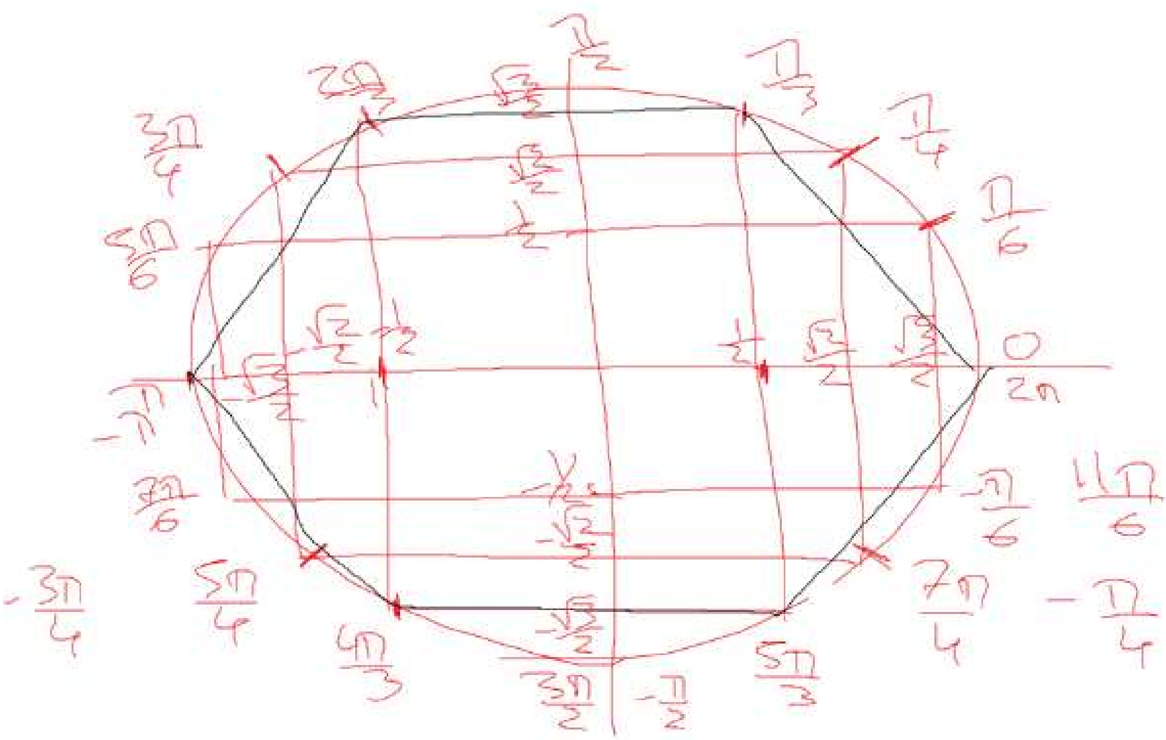
Propriété 7



$$\begin{cases} \cos(-x) = \cos x \\ \sin(-x) = -\sin x \end{cases} \qquad \begin{cases} \cos(\pi - x) = -\cos x \\ \sin(\pi - x) = \sin x \end{cases}$$

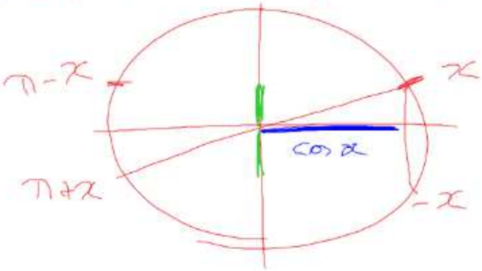
$$\begin{cases} \cos\left(\frac{\pi}{2}+x\right) = -\sin x \\ \sin\left(\frac{\pi}{2}+x\right) = \cos x \end{cases} \qquad \begin{cases} \cos(\pi + x) = -\cos x \\ \sin(\pi + x) = -\sin x \end{cases}$$

$$\begin{cases} \cos(x + 2\pi) = \cos x \\ \sin(x + 2\pi) = \sin x \end{cases} \qquad \begin{cases} \cos\left(\frac{\pi}{2}-x\right) = \sin x \\ \sin\left(\frac{\pi}{2}-x\right) = \cos x \end{cases}$$



Exemple 3

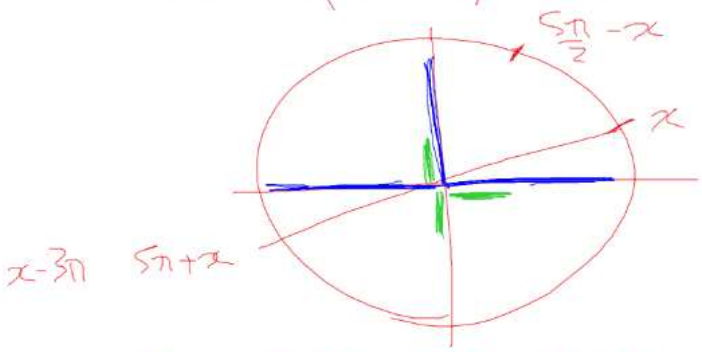
$A = \cos(-x) + \sin(-x) + \cos(\pi + x) + \sin(\pi - x)$



$A = \cos x - \sin x - \cos x + \sin x$

$A = 0$

$C = \sin(5\pi + x) + \cos(x - 3\pi) - \sin\left(\frac{5\pi}{2} - x\right)$



$C = -\sin x - \cos x - \cos x$

$= -\sin x - 2\cos x$

Example 4

1) $(\sin x - \cos x)^2 = \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= 1 - 2\sin x\cos x$

2) $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$\left(a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2\right)$

$= 1 - 2\sin^2 x\cos^2 x$

3) $(1 + \cos x + \sin x)^2 = 1 + \overbrace{\cos^2 x + \sin^2 x}^{1} + 2\cos x + 2\sin x + 2\cos x\sin x$

$= 2 + 2\cos x + 2\sin x + 2\cos x\sin x$

$= 2(1 + \cos x + \sin x + \cos x\sin x)$

$= 2(1 + \cos x)(1 + \sin x)$