

Ch 11: La fonction exponentielle.

Série 11.2.

Exemple 5

2) $f(x) = (2x-3)(1-e^{-x})$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} (2x-3) = +\infty \\ \lim_{x \rightarrow +\infty} e^{-x} = 0 \text{ donc } \lim_{x \rightarrow +\infty} (1-e^{-x}) = 1 \end{array} \right\} \begin{array}{l} \text{par produit} \\ \lim_{x \rightarrow +\infty} f(x) = +\infty \end{array}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} (2x-3) = -\infty \\ \lim_{x \rightarrow -\infty} e^{-x} = +\infty \text{ donc } \lim_{x \rightarrow -\infty} (1-e^{-x}) = -\infty \end{array} \right\} \begin{array}{l} \text{par produit} \\ \lim_{x \rightarrow -\infty} f(x) = +\infty \end{array}$$

4) $f(x) = x+3+xe^x$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} x = +\infty \\ \lim_{x \rightarrow +\infty} e^x = +\infty \end{array} \right\} \begin{array}{l} \text{par produit} \\ \lim_{x \rightarrow +\infty} xe^x = +\infty \end{array} \left. \begin{array}{l} \lim_{x \rightarrow +\infty} (x+3) = +\infty \end{array} \right\} \begin{array}{l} \text{par somme} \\ \lim_{x \rightarrow +\infty} f(x) = +\infty \end{array}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} xe^x = 0 \text{ par croissance comparée} \\ \lim_{x \rightarrow -\infty} (x+3) = -\infty \end{array} \right\} \begin{array}{l} \text{par somme} \\ \lim_{x \rightarrow -\infty} f(x) = -\infty \end{array}$$

6) $f(x) = \frac{e^x - 2}{e^x + 1}$

$$f(x) = \frac{e^x(1 - \frac{2}{e^x})}{e^x(1 + \frac{1}{e^x})} = \frac{1 - \frac{2}{e^x}}{1 + \frac{1}{e^x}}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} e^x = +\infty \text{ donc } \lim_{x \rightarrow +\infty} 1 - \frac{2}{e^x} = 1 \\ \lim_{x \rightarrow +\infty} 1 + \frac{1}{e^x} = 1 \end{array} \right\} \begin{array}{l} \text{par quotient} \\ \lim_{x \rightarrow +\infty} f(x) = 1 \end{array}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} e^x - 2 = -2 \\ \lim_{x \rightarrow -\infty} e^x + 1 = 1 \end{array} \right\} \begin{array}{l} \text{par quotient} \\ \lim_{x \rightarrow -\infty} f(x) = -2 \end{array}$$

$$3) f(x) = \frac{x-5}{e^x+1} = \frac{x(1-\frac{5}{x})}{e^x(1+\frac{1}{e^x})} = \frac{1-\frac{5}{x}}{\frac{e^x}{x}(1+\frac{1}{e^x})}$$

$$\lim_{x \rightarrow +\infty} (1 - \frac{5}{x}) = 1$$

$$\lim_{x \rightarrow +\infty} (1 + \frac{1}{e^x}) = 1$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x} = +\infty \text{ par croissance comparée}$$

} par quotient et produit
 $\lim_{x \rightarrow +\infty} f(x) = 0$

$$\lim_{x \rightarrow -\infty} (x-5) = -\infty$$

$$\lim_{x \rightarrow -\infty} (e^x+1) = 1$$

} par quotient $\lim_{x \rightarrow -\infty} f(x) = -\infty$

Exemple 6

$$2) f(x) = \frac{e^{5x}-1}{5x} \times \frac{5x}{3x} = \frac{e^{5x}-1}{5x} \times \frac{5}{3}$$

$$\lim_{x \rightarrow 0} \frac{e^{5x}-1}{5x} = \lim_{X \rightarrow 0} \frac{e^X-1}{X} \text{ en posant } X=5x$$

$$= 1$$

$$\text{donc } \lim_{x \rightarrow 0} f(x) = \frac{5}{3}$$

$$4) f(x) = e^x \frac{(e^{4x}-1)}{4x} = e^x \frac{e^{4x}-1}{4x} \times 2$$

$$\lim_{x \rightarrow 0} 2e^x = 2$$

$$\lim_{x \rightarrow 0} \frac{e^{4x}-1}{4x} = \lim_{X \rightarrow 0} \frac{e^X-1}{X} = 1 \text{ en posant } X=4x$$

$$\text{donc } \lim_{x \rightarrow 0} f(x) = 2$$

Example 7

2) $f_2(x) = (4x^2 - 2x + 3)e^x$

$$\begin{aligned}f_2'(x) &= (8x - 2)e^x + (4x^2 - 2x + 3)e^x \\&= e^x(8x - 2 + 4x^2 - 2x + 3) \\&= (4x^2 + 6x + 1)e^x\end{aligned}$$

4) $f_4(x) = \frac{e^x}{8e^x + 1}$

$$f_4'(x) = \frac{e^x(8e^x + 1) - e^x 8e^x}{(8e^x + 1)^2} = \frac{e^x}{(8e^x + 1)^2}$$

6) $f_6(x) = e^{3x^2 - 1}$

$$f_6'(x) = 6x e^{3x^2 - 1}$$

8) $f_8(x) = \frac{e^{-x}}{x^2}$

$$\begin{aligned}f_8'(x) &= \frac{-e^{-x}x^2 - 2xe^{-x}}{x^4} = \frac{-xe^{-x}(x + 2)}{x^4} \\&= \frac{-(x + 2)e^{-x}}{x^3}\end{aligned}$$