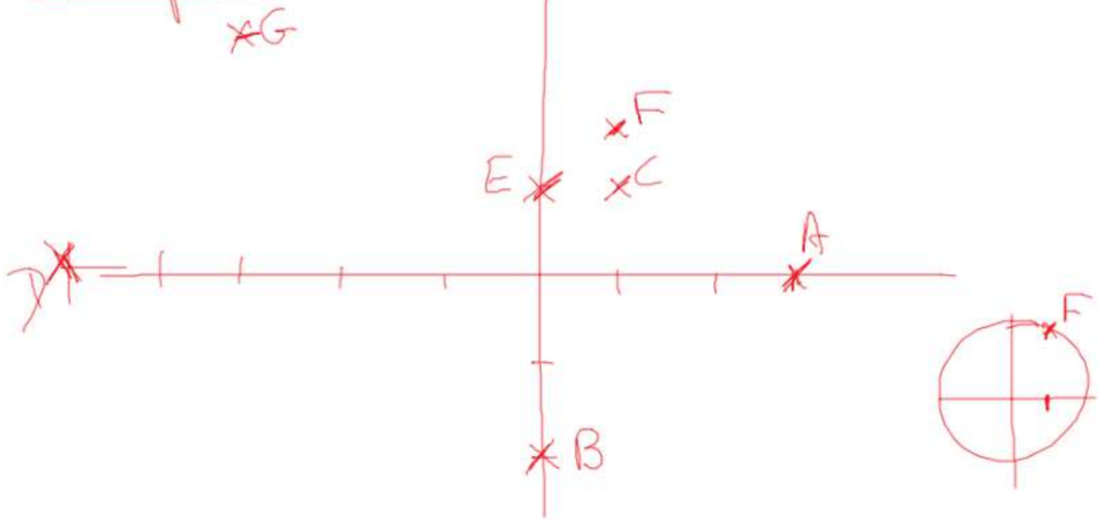
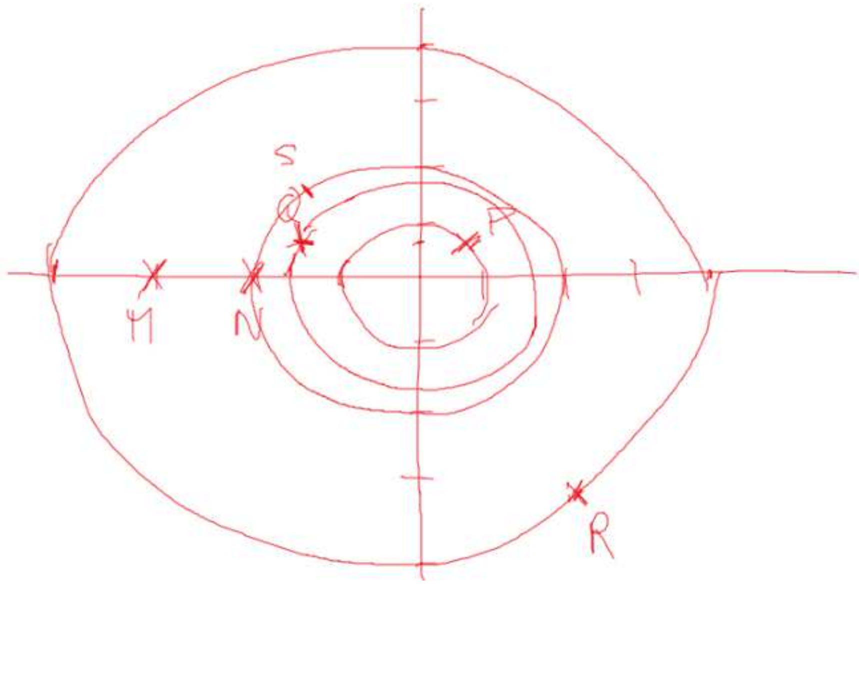
Exemples 1



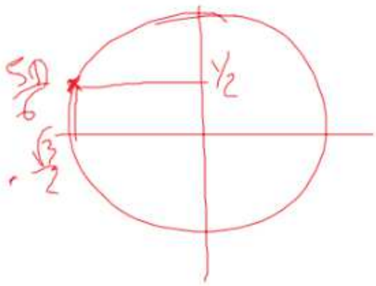
1) $A\left[3, 0\right]$ $B\left[2, \frac{3\pi}{2}\right]$ $C\left[\sqrt{2}, \frac{\pi}{4}\right]$

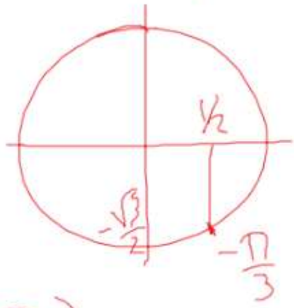
$D\left[5, \pi\right]$ $E\left[1, \frac{\pi}{2}\right]$ $F\left[2, \frac{\pi}{3}\right]$

$G\left[3\sqrt{2}, \frac{3\pi}{4}\right]$



$M(-3,0)$ $N(-2,0)$ $P\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ $Q\left(-\frac{3\sqrt{3}}{4},\frac{3}{4}\right)$





$R(2,-2\sqrt{3})$ $S(-\sqrt{2},\sqrt{2})$

Exemples 2

1] $z_1 = 1 + i$

Méthode calculatoire

$|z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

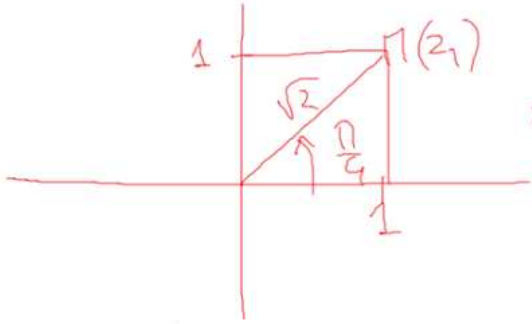
Soit θ un argument de z

$\left.\begin{array}{l} \cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\ \sin\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \end{array}\right\}$ donc $\theta = \frac{\pi}{4}$ convient

$\theta = \frac{\pi}{4} + k2\pi$ avec $k \in \mathbb{Z}$

Donc $z_1 = \left[\sqrt{2} ; \frac{\pi}{4}\right] = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$.

Méthode graphique



$z_1 = \left[\sqrt{2} ; \frac{\pi}{4}\right]$

3] $z_3 = -7 = [7, \pi] = 7(\cos\pi + i\sin\pi)$

5] $z_5 = -1 + i\sqrt{3}$

$|z_5| = \sqrt{(-1)^2 + \sqrt{3}^2} = \sqrt{4} = 2$

Soit θ un argument de z_5.

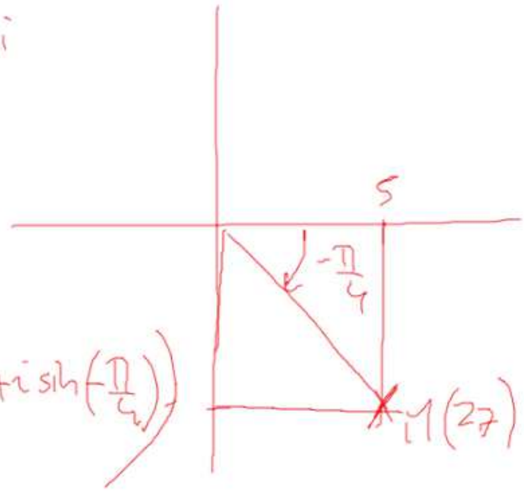
$\left.\begin{array}{l} \cos\theta = \dfrac{-1}{2} \\ \sin\theta = \dfrac{\sqrt{3}}{2} \end{array}\right\}$ donc $\theta = \frac{2\pi}{3}$ convient.

Donc $z_5 = \left[2 ; \frac{2\pi}{3}\right] = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

7) $z_7 = 5 - 5i$

$$z_7 = \left[5\sqrt{2} ; -\frac{\pi}{4}\right]$$

$$= 5\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right)$$



Exemples 3

1) $z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

$|z| = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

Soit θ un argument de z

$$\left.\begin{array}{l} \cos\theta = \dfrac{\frac{1}{2}}{1} = \dfrac{1}{2} \\ \sin\theta = \dfrac{-\frac{\sqrt{3}}{2}}{1} = -\dfrac{\sqrt{3}}{2} \end{array}\right\} \text{donc } \theta = -\frac{\pi}{3} \text{ convient}$$

donc $z = \left[1, -\frac{\pi}{3}\right]$

$$Z = z^{2018} = \left[1, -\frac{\pi}{3}\right]^{2018} = \left[1^{2018}, -\frac{2018\pi}{3}\right]$$

$$= \left[1, -\frac{2\pi}{3}\right]$$

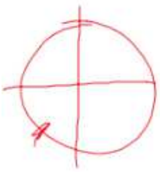
$$\begin{array}{r|l} 2018 & 6 \\ \cline{2-2} 21 & 336 \\ 38 & \\ 2 & \end{array}$$

$$= 1\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$

$2018 = 6 \times 336 + 2$

$2018\frac{\pi}{3} = \frac{6\pi}{3} \times 336 + \frac{2\pi}{3}$

$= 2\pi \times 336 + \frac{2\pi}{3}$



$$= -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

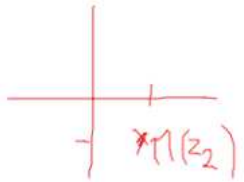
3] $z_1 = \sqrt{3} - i \qquad z_2 = 1 - i$ et $Z = \frac{z_1}{z_2}$

1) $|z_1| = \sqrt{3+1} = 2$

Soit θ_1 un argument de z_1

$$\left.\begin{array}{l} \cos\theta_1 = \frac{\sqrt{3}}{2} \\ \sin\theta_1 = \frac{-1}{2} \end{array}\right\} \text{donc } \theta_1 = -\frac{\pi}{6} \text{ convient}$$

Donc $z_1 = \left[2 ; -\frac{\pi}{6}\right]$



d'où $z_2 = \left[\sqrt{2}, -\frac{\pi}{4}\right]$

$$Z = \frac{z_1}{z_2} = \frac{\left[2, -\frac{\pi}{6}\right]}{\left[\sqrt{2}, -\frac{\pi}{4}\right]} = \left[\frac{2}{\sqrt{2}}, -\frac{\pi}{6} + \frac{\pi}{4}\right]$$

$$= \left[\sqrt{2}, \frac{\pi}{12}\right]$$

$$= \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

b) $Z = \frac{z_1}{z_2} = \frac{\sqrt{3} - i}{1 - i} = \frac{(\sqrt{3} - i)(1 + i)}{(1 - i)(1 + i)}$

$= \frac{\sqrt{3} + \sqrt{3}i - i + 1}{2}$

$= \frac{1 + \sqrt{3}}{2} + i\,\frac{\sqrt{3} - 1}{2}$

On en déduit que :

$\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}$

Donc $\sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2}$ et $\sqrt{2}\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2}$

Ainsi $\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$

$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}.$