Chapitre 13 : Suites (1)

Série 13 - 1

Exemple 1

2)
$$u_1 = 1 - \frac{2}{1} = 1 - 2 = -1$$
$$u_2 = 2 - \frac{2}{2} = 2 - 1 = 1$$
$$u_3 = 3 - \frac{2}{3} = \frac{7}{3}$$
$$u_4 = 4 - \frac{2}{4} = 4 - \frac{1}{2} = \frac{7}{2}$$

4)
$$u_1 = \frac{5u_0 - 3}{u_0 + 1} = \frac{10-3}{2+1} = \frac{7}{3}$$
$$u_2 = \frac{5u_1 - 3}{u_1 + 1} = \frac{5\times\frac{7}{3} - 3}{\frac{7}{3}+1} = \frac{\frac{35}{3} - \frac{9}{3}}{\frac{10}{3}} = \frac{\frac{26}{3}}{\frac{10}{3}} = \frac{26}{10} = \frac{13}{5}$$
$$u_3 = \frac{5u_2 - 3}{u_2 + 1} = \frac{5\times\frac{13}{5} - 3}{\frac{13}{5}+1} = \frac{13 - 3}{\frac{18}{5}} = 10 \times \frac{5}{18} = \frac{25}{9}$$
$$u_4 = \frac{5u_3 - 3}{u_3 + 1} = \frac{5\times\frac{25}{9} - 3}{\frac{25}{9}+1} = \frac{\frac{125-27}{9}}{\frac{34}{9}} = \frac{\frac{98}{9}}{\frac{34}{9}} = \frac{98}{34} = \frac{49}{17}$$

5)
$$u_3 = u_2 - \frac{2}{2\times(2-1)} = 4 - \frac{2}{2\times1} = 4 - 1 = 3$$
$$u_4 = u_3 - \frac{2}{3\times(3-1)} = 3 - \frac{2}{3\times2} = 3 - \frac{1}{3} = \frac{8}{3}$$
$$u_5 = u_4 - \frac{2}{4\times(4-1)} = \frac{8}{3} - \frac{2}{4\times3} = \frac{8}{3} - \frac{1}{6} = \frac{15}{6} = \frac{5}{2}$$
$$u_6 = u_5 - \frac{2}{5\times(5-1)} = \frac{5}{2} - \frac{2}{5\times4} = \frac{5}{2} - \frac{1}{10} = \frac{24}{10} = \frac{12}{5}$$

6)
$$u_2 = u_1 + \frac{1}{3} - \frac{1}{3\times1(1+1)} = \frac{2}{3} + \frac{1}{3} - \frac{1}{6} = 1 - \frac{1}{6} = \frac{5}{6}$$
$$u_3 = u_2 + \frac{1}{3} - \frac{1}{3\times2\times3} = \frac{5}{6} + \frac{1}{3} - \frac{1}{18} = \frac{15+6-1}{18} = \frac{20}{18} = \frac{10}{9}$$
$$u_4 = u_3 + \frac{1}{3} - \frac{1}{3\times3\times4} = \frac{10}{9} + \frac{1}{3} - \frac{1}{36} = \frac{40+12-1}{36} = \frac{51}{36} = \frac{17}{12}$$
$$u_5 = u_4 + \frac{1}{3} - \frac{1}{3\times4\times5} = \frac{17}{12} + \frac{1}{3} - \frac{1}{60} = \frac{85+20-1}{60} = \frac{104}{60} = \frac{26}{15}$$

Exemple 2

2] $u_n = n^2 + n - 2$

$u_{n+1} = (n+1)^2 + n+1-2 = n^2+2n+1+n-1 = n^2+3n$

$u_n + 1 = n^2+n-2+1 = n^2+n-1$

$u_{2n} = (2n)^2 + (2n) - 2 = 4n^2 + 2n - 2$

$u_{3n-1} = (3n-1)^2 + (3n-1) - 2 = 9n^2 - 6n + 1 + 3n - 1 - 2 = 9n^2 - 3n - 2$

$u_{n+1} - u_n = n^2 + 3n - (n^2 + n - 2) = 2n + 2$

3] $v_{n+1} = 2^{3(n+1)+1} = 2^{3n+4}$

$v_{n-1} = 2^{3(n-1)+1} = 2^{3n-2}$

$v_{n+2} = 2^{3(n+2)+1} = 2^{3n+7}$

$v_{2n} = 2^{3(2n)+1} = 2^{6n+1}$

$v_{2n+1} = 2^{3(2n+1)+1} = 2^{6n+4}$

4] $w_{n+1} = 3^{2(n+1)-1} = 3^{2n+1}$

$w_{n-1} = 3^{2(n-1)-1} = 3^{2n-3}$

$w_{n+2} = 3^{2(n+2)-1} = 3^{2n+3}$

$w_{2n} = 3^{2(2n)-1} = 3^{4n-1}$

$w_{2n+1} = 3^{2(2n+1)-1} = 3^{4n+1}$

6] $y_{n+1} = 3y_n + 1$

$y_{n+2} = 3y_{n+1} + 1$

$= 3(3y_n + 1) + 1$

$= 9y_n + 4$

Exemple 3

b) $u_{n+1} - u_n = n^2 \geqslant 0.$

La suite (u_n) est croissante.

d) Méthode 1

$$u_{n+1} - u_n = \frac{2}{n+1} - \frac{2}{n} = \frac{2n - 2(n+1)}{n(n+1)} = \frac{-2}{n(n+1)}$$

Comme $n \in \mathbb{N}$, $n(n+1) \geqslant 0$ et donc $\frac{-2}{n(n+1)} \leqslant 0$.

La suite (u_n) est décroissante.

Méthode 2

$u_n = f(n)$ avec $f(x) = \frac{2}{x}$ pour $x \in \mathbb{R}^{+*}$

On a : $f'(x) = -\frac{2}{x^2} < 0$. La fonction f est décroissante sur $\mathbb{R}^{+*}$

Donc la suite (u_n) est décroissante.

f) $u_1 = -1$

$u_2 = \frac{(-1)^2}{2} = \frac{1}{2}$

$u_3 = \frac{(-1)^3}{3} = \frac{-1}{3}$

donc $u_1 \leqslant u_2$

La suite (u_n) n'est pas décroissante.

donc $u_2 > u_3$

La suite (u_n) n'est pas croissante.

La suite (u_n) n'est ni croissante, ni décroissante.

h) $$u_{n+1} - u_n = \left(\frac{2}{3}\right)^{n+1} - \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^n \times \left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^n$$

$$= \left(\frac{2}{3}\right)^n \left(\frac{2}{3} - 1\right) = \left(\frac{2}{3}\right)^n \times \left(\frac{-1}{3}\right)$$

Donc $u_{n+1} - u_n < 0$.

La suite (u_n) est décroissante