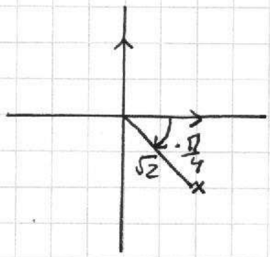
Chapitre 15 : Nombres complexes (2)

Série 15-3.

<u>Exemple 5</u>

4) • $z_1 = 2(1-i)$.



Graphiquement, on voit que : $1-i = \sqrt{2}\, e^{-\frac{i\pi}{4}}$

Donc $z_1 = 2 \times \sqrt{2} e^{-i\pi/4} = 2\sqrt{2}\, e^{-i\pi/4}$

• On sait que : $-1 = e^{i\pi}$ et $i = e^{i\pi/2}$

Donc $z_2 = \frac{2}{5} \times (-1) \times i = \frac{2}{5} e^{i\pi} e^{\frac{i\pi}{2}} = \frac{2}{5} e^{\frac{3i\pi}{2}}$

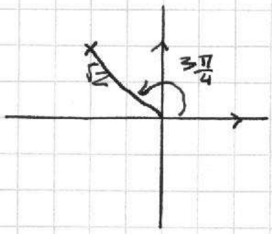
• On a : $1+\sqrt{3} \in \mathbb{R}^{+*}$

Donc $1+\sqrt{3} = (1+\sqrt{3})\, e^{i0}$

• On sait que : $-1 = e^{i\pi}$ et $i = e^{\frac{i\pi}{2}}$

Donc $z_4 = \sqrt{2} \times (-1) \times i = \sqrt{2}\, e^{i\pi} e^{i\pi/2} = \sqrt{2}\, e^{\frac{3i\pi}{2}}$

• $z_5 = 3(-1+i)$



Graphiquement, on voit que : $-1+i = \sqrt{2}\, e^{\frac{3i\pi}{4}}$

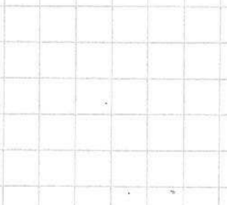
Donc $z_5 = 3\sqrt{2} e^{\frac{3i\pi}{4}}$

• $z_6 = -5(1-i) = 5 \times (-1)(1-i)$

Or $-1 = e^{i\pi}$ et $1-i = \sqrt{2} e^{-\frac{i\pi}{4}}$ (voir z_1)

Donc $z_6 = 5 e^{i\pi} \sqrt{2} e^{-\frac{i\pi}{4}} = 5\sqrt{2}\, e^{\frac{3i\pi}{4}}$

• $z_7 = (1+i)(1-\sqrt{3}i)$



Graphiquement, on voit que : $1+i = \sqrt{2} e^{i\pi/4}$

$|1-\sqrt{3}i| = \sqrt{1+(-\sqrt{3})^2} = \sqrt{4} = 2$

Soit θ un argument de $1-\sqrt{3}i$

$$\left.\begin{array}{l} \cos\theta = \frac{1}{2} \\ \sin\theta = -\frac{\sqrt{3}}{2} \end{array}\right\} \text{ donc } \theta = -\frac{\pi}{3} \text{ convient.}$$

Ainsi $1-\sqrt{3}i = 2\, e^{-\frac{i\pi}{3}}$

Donc $z_7 = \sqrt{2} e^{i\pi/4} \times 2 e^{-i\pi/3} = 2\sqrt{2}\, e^{\frac{3i\pi}{12} - \frac{4i\pi}{12}}$

$= 2\sqrt{2} e^{-\frac{i\pi}{12}}$

- $z_8 = \dfrac{1-i}{\sqrt{3}+i}$

Graphiquement, on voit que : $1-i = \sqrt{2}\,e^{-i\frac{\pi}{4}}$ (voir z_1)

$|\sqrt{3}+i| = \sqrt{\sqrt{3}^2+1^2} = 2$

Soit θ un argument de $\sqrt{3}+i$

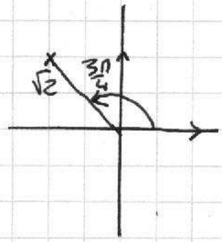
$$\left.\begin{array}{l}\cos\theta = \dfrac{\sqrt{3}}{2} \\ \sin\theta = \dfrac{1}{2}\end{array}\right\} \text{ donc } \theta = \frac{\pi}{6} \text{ convient.}$$

Ainsi $\sqrt{3}+i = 2e^{i\pi/6}$.

Donc $z_8 = \dfrac{\sqrt{2}\,e^{-i\pi/4}}{2\,e^{i\pi/6}} = \dfrac{\sqrt{2}}{2}\,e^{-i\pi/4 - i\pi/6} = \dfrac{\sqrt{2}}{2}\,e^{-5i\frac{\pi}{12}}$

<u>Exemple 6</u>

2] a) $z_1 = 3(-1+i)$



Graphiquement, on voit que : $-1+i = \sqrt{2}\,e^{3i\frac{\pi}{4}}$

Donc $z_1 = 3\sqrt{2}\,e^{3i\frac{\pi}{4}}$

$z_2 = 5(1-\sqrt{3}i)$

$|1-\sqrt{3}i| = \sqrt{1^2+(-\sqrt{3})^2} = \sqrt{4} = 2$

Soit θ un argument de $1-\sqrt{3}i$

$$\left.\begin{array}{l}\cos\theta = \dfrac{1}{2} \\ \sin\theta = -\dfrac{\sqrt{3}}{2}\end{array}\right\} \text{ donc } \theta = -\frac{\pi}{3} \text{ convient.}$$

Ainsi $1-\sqrt{3}i = 2e^{-i\frac{\pi}{3}}$

Donc $z_2 = 5\times 2e^{-i\frac{\pi}{3}} = 10e^{-i\frac{\pi}{3}}$

b) $\overline{z_1} = 3\sqrt{2}\,e^{-3i\frac{\pi}{4}}$

$\overline{z_2} = 10\,e^{i\frac{\pi}{3}}$

$z_1 z_2 = 3\sqrt{2}\,e^{3i\frac{\pi}{4}} \times 10e^{-i\frac{\pi}{3}} = 30\sqrt{2}\,e^{i\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)}$

$= 30\sqrt{2}\,e^{5i\frac{\pi}{12}}$

$\dfrac{z_1}{z_2} = \dfrac{3\sqrt{2}\,e^{3i\frac{\pi}{4}}}{10\,e^{-i\frac{\pi}{3}}} = \dfrac{3\sqrt{2}}{10}\,e^{i\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)}$

$= \dfrac{3\sqrt{2}}{10}\,e^{13i\frac{\pi}{12}}$

c) $z_1^{10} = \left(3\sqrt{2}\, e^{\frac{3i\pi}{4}}\right)^{10} = (3\sqrt{2})^{10}\, e^{\frac{30i\pi}{4}}$

$= 1889568\, e^{-i\frac{\pi}{2}}$

$z_2^{4} = \left(10 e^{-i\frac{\pi}{3}}\right)^{4} = 100^2 e^{-4i\frac{\pi}{3}}$

$= 10000\, e^{2i\frac{\pi}{3}}$

4) $|1+\sqrt{3}i| = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$

soit θ un argument de $1+\sqrt{3}i$

$\left.\begin{array}{l} \cos\theta = \frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}}{2} \end{array}\right\}$ donc $\theta = \frac{\pi}{3}$ convient.

Ainsi $1+\sqrt{3}i = 2e^{i\frac{\pi}{3}}$

D'où $1-\sqrt{3}i = \overline{1+\sqrt{3}i} = 2e^{-i\frac{\pi}{3}}$

Donc $= \dfrac{(2e^{i\pi/3})^2}{(2e^{-i\frac{\pi}{3}})^3} - \dfrac{(2e^{-i\pi/3})^2}{(2e^{i\pi/3})^3}$

$= \dfrac{4e^{2i\pi/3}}{8e^{-i\pi}} - \dfrac{4e^{-2i\frac{\pi}{3}}}{8e^{i\pi}}$

$= \dfrac{4e^{2i\frac{\pi}{3}}}{-8} - \dfrac{4e^{-2i\pi/3}}{-8}$

$= -\frac{1}{2} e^{2i\frac{\pi}{3}} + \frac{1}{2} e^{-2i\frac{\pi}{3}}$

$= \frac{1}{2}\left(e^{-2i\frac{\pi}{3}} - e^{2i\frac{\pi}{3}}\right)$

$= \frac{1}{2}\left(\cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3}) - \cos(\frac{2\pi}{3}) - i\sin(\frac{2\pi}{3})\right)$

$= \frac{1}{2}\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2} + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$

$= -\frac{i\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} \times (-1) \times i$

$= \frac{\sqrt{3}}{2} \times e^{i\pi} \times e^{i\pi/2}$

$= \frac{\sqrt{3}}{2} e^{3i\frac{\pi}{2}}$