TD n° 15 Nombres complexes (2)

Exercice 1

$|z_1| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|z_2| = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|z_3| = \frac{2}{3}$

$|z_4| = \left|-\frac{4}{7}\right| \times |i| = \frac{4}{7} \times 1 = \frac{4}{7}$

$|z_5| = \sqrt{\sqrt{2}^2+1^2} = \sqrt{3}$

$|z_6| = \sqrt{\sqrt{2}^2+(-\sqrt{3})^2} = \sqrt{2+3} = \sqrt{5}$

$|z_7| = |5-3i| \times |12+7i| = \sqrt{5^2+(-3)^2} \times \sqrt{12^2+7^2} = \sqrt{25+9} \times \sqrt{144+49}$

$= \sqrt{34} \times \sqrt{193} = \sqrt{6562}$

$|z_8| = |3-4i|^5 = \left(\sqrt{3^2+(-4)^2}\right)^5 = \left(\sqrt{9+16}\right)^5 = \left(\sqrt{25}\right)^5 = 5^5 = 3125$

$|z_9| = \frac{3}{(2+i)^2} = \frac{|3|}{|2+i|^2} = \frac{3}{\left(\sqrt{4+1}\right)^2} = \frac{3}{\sqrt{5}^2} = \frac{3}{5}$

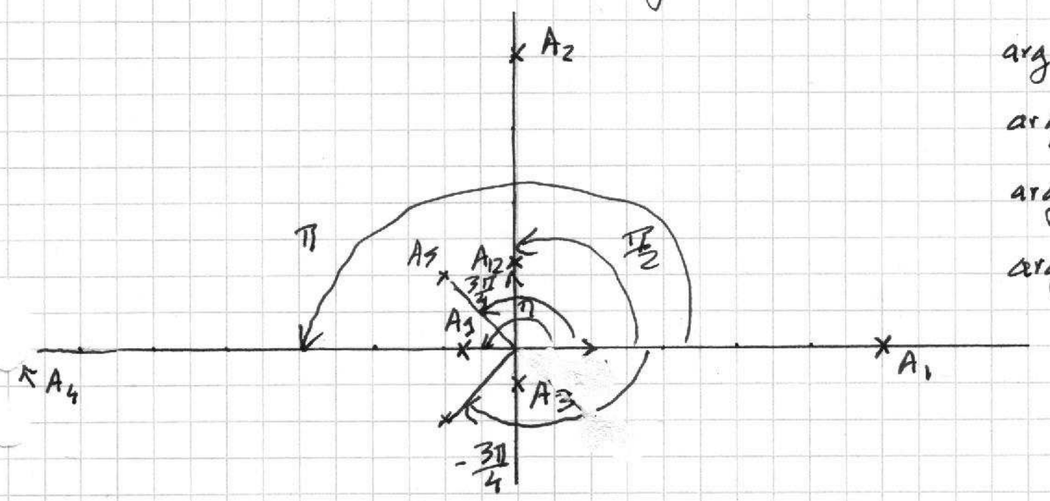
$|z_{10}| = \frac{|2+i|^3}{|1-i|^2} = \frac{\left(\sqrt{2^2+1^2}\right)^3}{\left(\sqrt{1+(-1)}\right)^2} = \frac{\sqrt{5}^3}{\sqrt{2}^2} = \frac{5\sqrt{5}}{2}$

$|z_{11}| = \left|\frac{a+b}{iab}\right| = \frac{|a+b|}{|iab|} = \frac{|a+b|}{|i|\,|ab|} = \frac{|a+b|}{1 \times |ab|} = \frac{|a+b|}{|ab|}$

$|z_{12}| = \frac{|a-ib|}{|a+ib|} = \frac{\sqrt{a^2+(-b)^2}}{\sqrt{a^2+b^2}} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} = 1$

Exercice 2

Si on note A_1, A_2, A_3, A_4 et A_5 les points image de z_1, z_2, z_3 et z_5 on obtient aussitôt un argument.



$\arg(z_1) = 0$

$\arg(z_2) = \frac{\pi}{2}$

$\arg(z_3) = -\frac{\pi}{2}$

$\arg(z_4) = \pi$

$\arg(z_5) = \frac{3\pi}{4}$

- $|z_6| = \sqrt{\sqrt{3}^2+1^2} = \sqrt{4} = 2$

  Soit θ un argument de z_6.

  $$\left.\begin{array}{l}\cos\theta = \frac{\sqrt{3}}{2}\\ \sin\theta = \frac{1}{2}\end{array}\right\}$$ donc $\theta = \frac{\pi}{6}$ convient. Donc $\arg(z_6) = \frac{\pi}{6}$

- $\arg(z_7) = \arg(\sqrt{2}(1-i)) = \arg\sqrt{2} + \arg(1-i) = 0 + (-\frac{\pi}{4}) = -\frac{\pi}{4}$
- $|z_8| = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{1+3} = 2.$

  Soit θ un argument de z_8

  $$\left.\begin{array}{l}\cos\theta = \frac{-1}{2}\\ \sin\theta = -\frac{\sqrt{3}}{2}\end{array}\right\}$$ donc $\theta = -\frac{2\pi}{3}$ convient. Donc $\arg(z_8) = -\frac{2\pi}{3}$

- On note A_9 le point image de z_9.

  D'après le graphique, $\arg(z_9) = \pi$
- $\arg(z_{10}) = \arg(5(1+i)) = \arg 5 + \arg(1+i) = 0 + \frac{\pi}{4} = \frac{\pi}{4}$
- On note A_{11} le point image de z_{11}.

  D'après le graphique, $\arg(z_{11}) = -\frac{3\pi}{4}$
- On note A_{12} le point image de z_{12}

  D'après le graphique, $\arg(z_{12}) = \frac{\pi}{2}$.

<u>Exercice 3</u>

1] $\arg(z) = 0 \iff z \in \mathbb{R}^{+*}$

$\arg(z) = \frac{\pi}{2} \iff z \in i\mathbb{R}^{+*}$

$\arg(z) = -\frac{\pi}{2} \iff z \in i\mathbb{R}^{-*}$

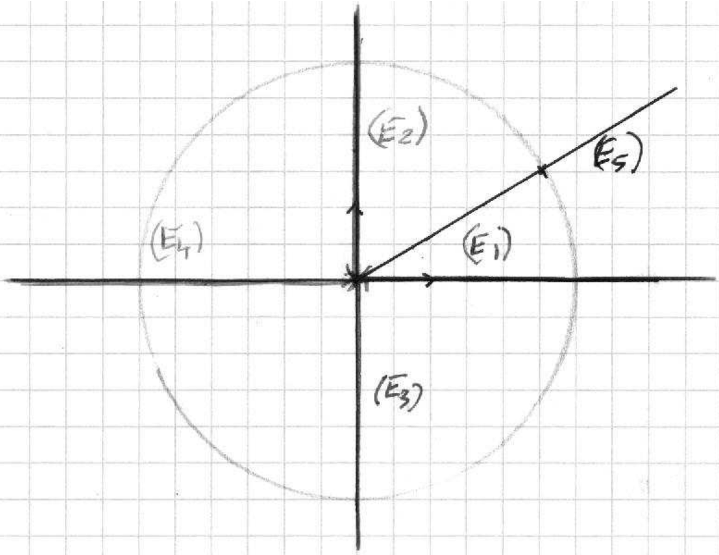
$\arg(z) = \pi \iff z \in \mathbb{R}^{-*}$

$\arg(z) = \frac{\pi}{6} \iff \arg(z) = \arg(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$\iff \arg(z) = \arg(\frac{\sqrt{3}}{2} + i\frac{1}{2})$

$\iff z = k(\frac{\sqrt{3}}{2} + i\frac{1}{2})$ avec $k \in \mathbb{R}^{+*}$

On note (E_1), (E_2), (E_3), (E_4) et (E_5) les représentations respectives de ces 5 équations.



2] $\arg(\bar{z}) = \frac{\pi}{6} \iff -\arg(z) = \frac{\pi}{6}$

$\iff \arg(z) = -\frac{\pi}{6}$

$\iff \arg(z) = {}^{\arg}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

$\iff z = k\left(+\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$ avec $k \in \mathbb{R}^{+*}$

On note (E) l'ensemble des points M correspondants

3] $\arg(-3z) = \frac{\pi}{2} \iff \arg(-3) + \arg(z) = \frac{\pi}{2}$

$\iff \pi + \arg(z) = \frac{\pi}{2}$

$\iff \arg(z) = -\frac{\pi}{2}$

$\iff z \in i\mathbb{R}^{-*}$

On note (F) l'ensemble des points M correspondants.